

Tools for tiling surfaces

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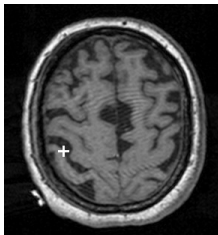


- 1 Introduction
 - Surfaces
 - Shape description
- 2 Cutting / tiling
 - Cutting: why?
 - Surface cutting
 - Tiling a surface
- 3 Combine geometry and topology
 - Non-Euclidean distances
 - Medical imaging application
 - Selecting the extrema
 - Computer graphics application
- 4 Non trivial tiles
 - Cutting into cylinders
 - n -cets
 - Cutting by n -loops
 - Applications
- 5 Conclusion
 - Synthesis
 - Future



Motivations

Amount of **available data**

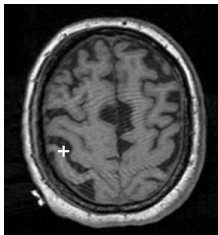


- Various acquisitions (medical imaging, 3D scanners)
- Modeling (computer graphics, Computer-aided design)

Sources: AIM@Shape, Stanford 3D Scanning Repository

Motivations

Amount of **available data**



Various problems:

- Analysis and measurement
- Transformations, distortions

Sources: AIM@Shape, Stanford 3D Scanning Repository

Geometry

Various scale levels of local information



Quantification: local curvatures, distances, patterns, ...

Geometry

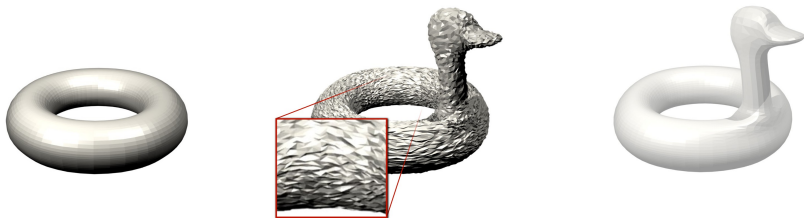
Various scale levels of local information



Quantification: local curvatures, distances, patterns, ...

Geometry

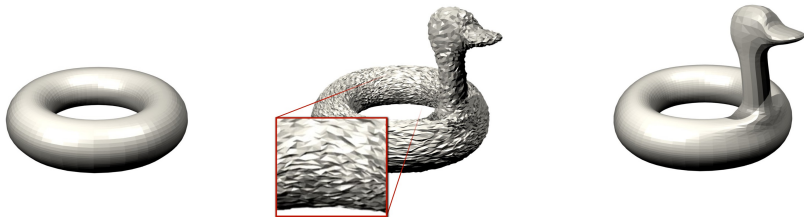
Various scale levels of local information



Quantification: local curvatures, distances, patterns, ...

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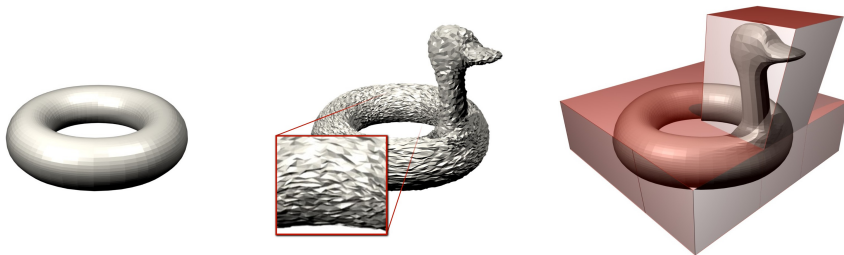
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Quantification: local curvatures, distances, patterns, ...

Geometry

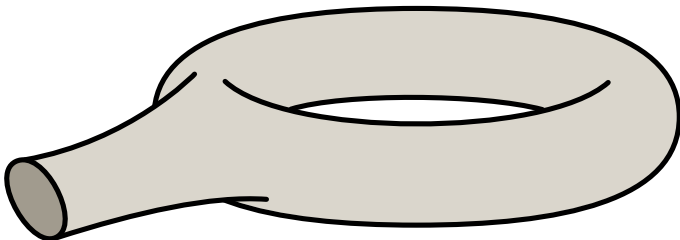
Various scale levels of local information



Quantification: local curvatures, distances, patterns, ...

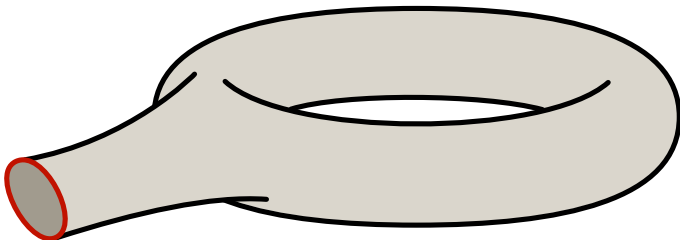
Topology

Global information



Topology

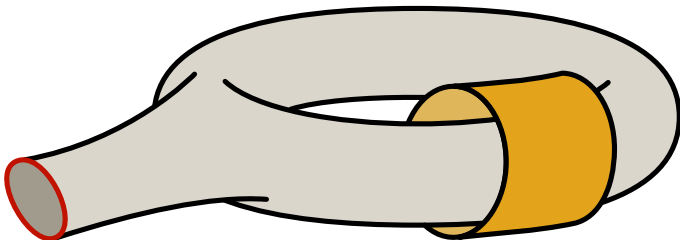
Global information



- Number of **boundaries**

Topology

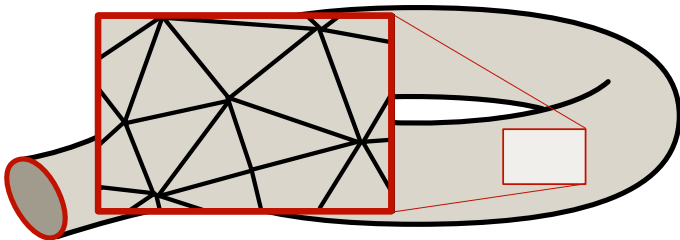
Global information



- Number of **boundaries**
- Number of handles: **genus**

Simplicial complexes

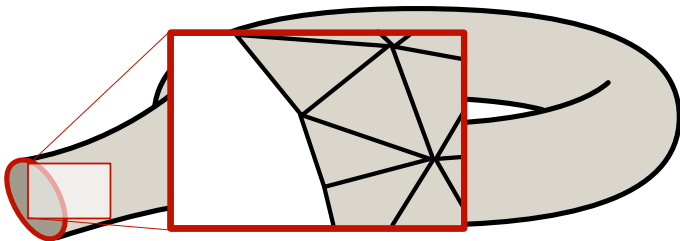
Discrete representation by 2-meshes



- **Disc:** non-boundary region

Simplicial complexes

Discrete representation by 2-meshes



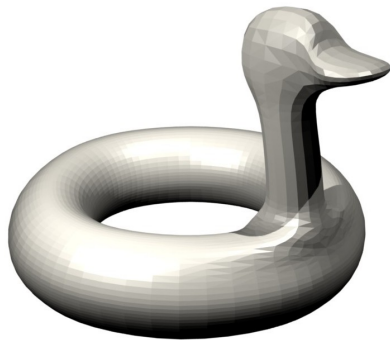
- **Disc:** non-boundary region
- **Half-disc:** boundary region

Manipulating surfaces

Implies using both **geometry** and **topology**

- Topological tools: **global**
- Geometrical components: **local**

Towards a global using of the geometry



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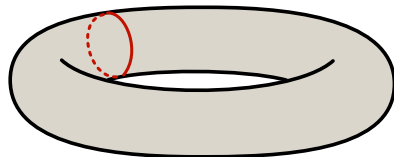
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Cutting: why?

Cutting reasons

- Topological changes



Cutting: why?

Cutting reasons

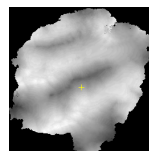
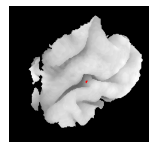
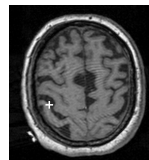
- Topological changes
- Segmentation



Cutting: why?

Cutting reasons

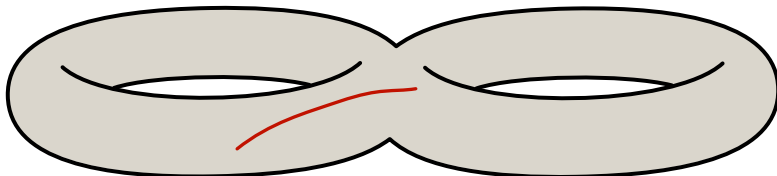
- Topological changes
- Segmentation
- Facilitate following manipulations



Paths, loops

Cutting according to:

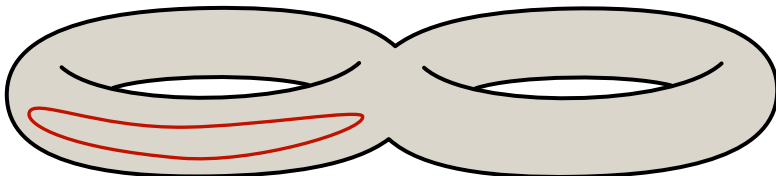
- a path



Paths, loops

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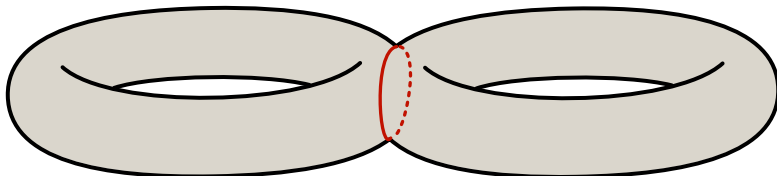
- a path
- a loop



Paths, loops

Cutting according to:

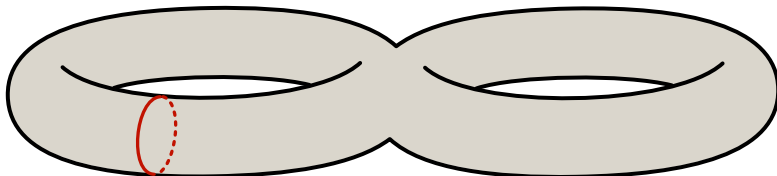
- a path
- a loop



Paths, loops

Cutting according to:

- a path
- a non-separating loop

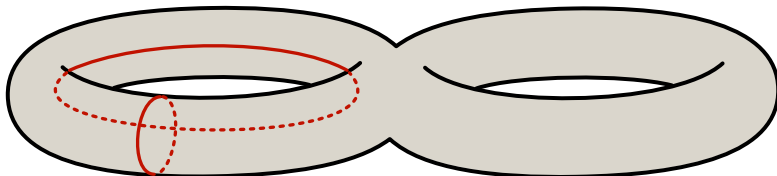


⇒ **Reducing** the genus

Paths, loops

Definition

A **cutting** on a 2-mesh \mathcal{M} is defined as a set of paths and loops $(C_i)_{1 \leq i \leq n}$



⇒ **Reducing** the genus and the number of boundaries

Topological cutting

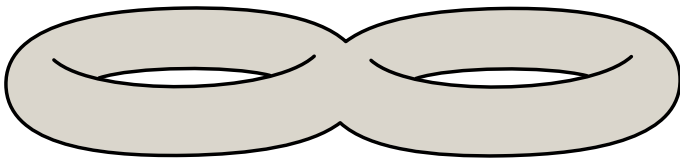
Optimal cutting — variation

Input: a surface \mathcal{M} (non homeomorphic to a disc)

Begin

- **While** $\text{genus}(\mathcal{M}) \neq 0$ **do**
 - Find the **shortest** non separating **loop** l
 - Cut \mathcal{M} according to l
- Cut according to the **shortest spanning tree** joining the boundaries of \mathcal{M}

End



Genus: 2 — boundaries: 0

J. Erickson and S. Har-Peled, *Optimally Cutting a Surface into a Disk*, ACM Symposium on Computational Geometry, 2002

Topological cutting

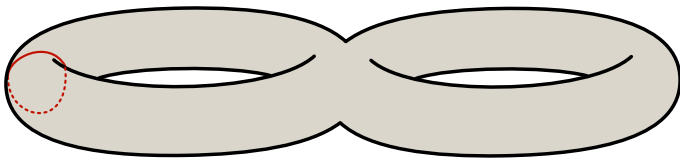
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Genus: 1 — boundaries: 2

J. Erickson and S. Har-Peled, *Optimally Cutting a Surface into a Disk*, ACM Symposium on Computational Geometry, 2002

Topological cutting

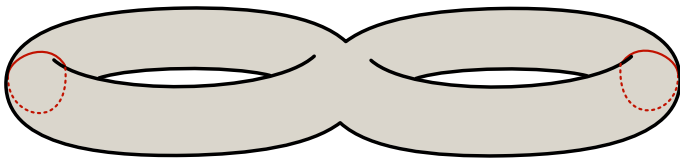
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End



Genus: 0 — boundaries: 4

J. Erickson and S. Har-Peled, *Optimally Cutting a Surface into a Disk*, ACM Symposium on Computational Geometry, 2002

Topological cutting

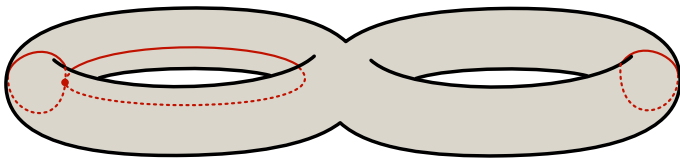
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End



Genus: 0 — boundaries: 3

J. Erickson and S. Har-Peled, *Optimally Cutting a Surface into a Disk*, ACM Symposium on Computational Geometry, 2002

Topological cutting

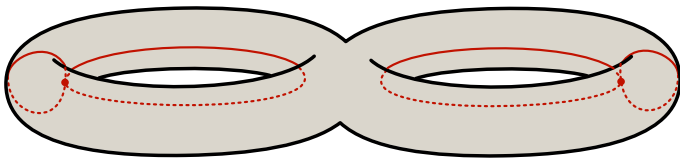
Optimal cutting — variation

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End



Genus: 0 — boundaries: 2

J. Erickson and S. Har-Peled, *Optimally Cutting a Surface into a Disk*, ACM Symposium on Computational Geometry, 2002

Topological cutting

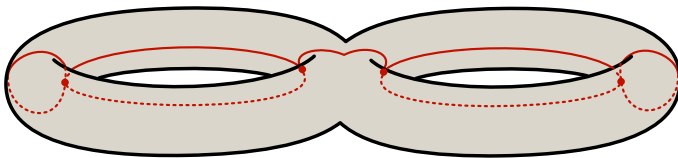
Optimal cutting — variation

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End



Genus: 0 — boundaries: 1

J. Erickson and S. Har-Peled, *Optimally Cutting a Surface into a Disk*, ACM Symposium on Computational Geometry, 2002

Computation time, topological cutting

Surface		Algorithme exact		Algorithme approché	
Genus	# vertices	Time (s)	Length	Time (s)	Length
1	3,072	0.51	3.139,33	0.04	3.245,53
1	12,288	5.09	3.139,33	0.19	3.180,83
2	3,326	1.05	7.070,2	0.08	7.676,53
2	13,310	9.72	7.070,2	0.42	7.392,49
3	6,908	3.45	34.360,9	0.34	55.308,7
3	27,644	32.49	34.360,9	1.84	37.619
4	8,826	5.48	41.770,3	0.68	83.485,4
4	35,322	58.37	41.770,3	3.69	66.257,5
104	543,652	—	—	48,mn	—

Comparison of an exact method and an approximative cutting method using shortest non separating loops.

Tiling



Classical definition:

- On the plane
- **Repeated** patterns (tiles)
- Topologically trivial

Figure: M. C. Escher

Tiling



In our context:

- On surfaces
- Topology of the tiles not fixed
- Need for a **precise description**

Figure: tiling by discs and cylinders

m-cellular complex

Definition

A m -cellular complex \mathcal{C} is a set of m -cells with :

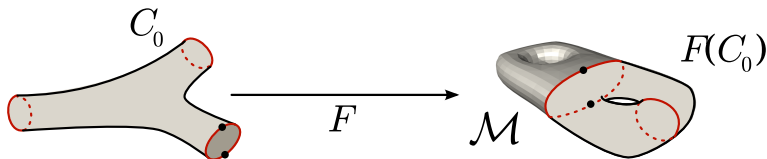
- 2- m -cells: **compact connex 2-manifold** with boundary
- 1- m -cells: **simple curves** (closed or opened)
- 0- m -cells: **points**
- **boundary** of any i - m -cell: $(i - 1)$ -sub-complex
- The **intersection** of two m -cells is an m -cell of \mathcal{C}



Tile

$F : C \longrightarrow \mathcal{M}$ embedding of a 2-m-cell C in a 2-mesh \mathcal{M} , with:

- F continuous.
- v **vertex** of $C \Rightarrow F(v)$ **0-simplex** of \mathcal{M} .
- e **edge** of $C \Rightarrow F(e)$ **set of 1-simplexes** de \mathcal{M} .
- b **body** of $C \Rightarrow F(b)$ **set of 2-simplexes** de \mathcal{M} .
- F **injective** on the interior of C . Images of two edges or two vertices should be equal.

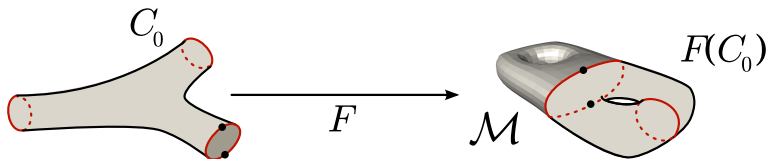


Tile

Equivalence relation

$F \sim F'$ iff for all cell, edge or vertex X of C , $F(X) = F'(X)$.

M-tiles = equivalence classes

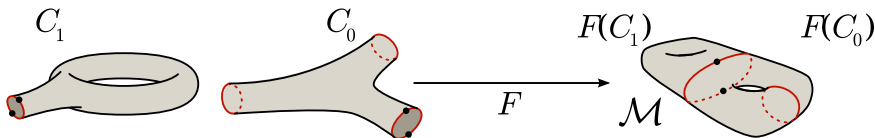


Tiling

M-tiling defined by equivalence classes of embeddings $F : \mathcal{C} \longrightarrow \mathcal{M}$ where:

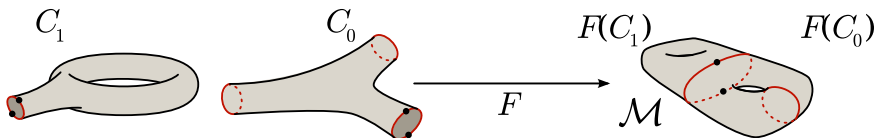
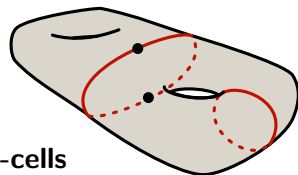
- \forall 2-m-cell C of \mathcal{C} , $F|_C$ is an **M-tile**
- F **injective** except on the boundaries
- F is **surjective**

Two embeddings F and F' are equivalents $\Leftrightarrow \forall X \in \mathcal{C}$, $F(X) = F'(X)$.



Cutting/tiling duality

- **Paths and loops:** images of the 1- m -cells
- **Extremities** of the paths: images of the 0- m -cells
- **Complement:** images of the 2- m -cells

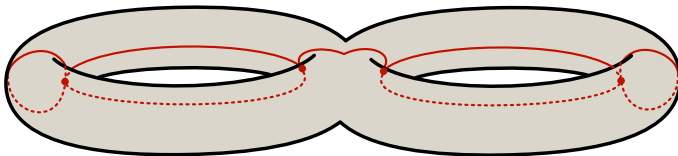


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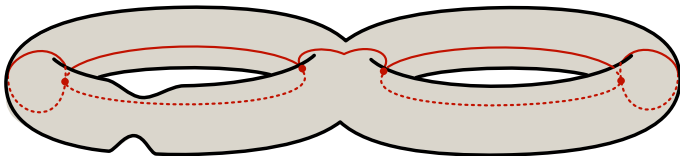
Geometry as an component in the cutting process

General structure of the algorithms: **topology**



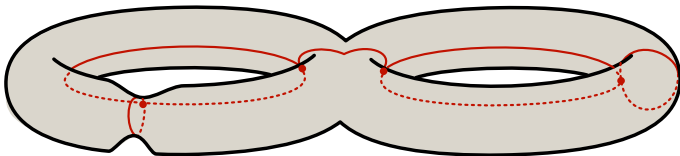
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Geometry as an component in the cutting process

General structure of the algorithms: **topology**

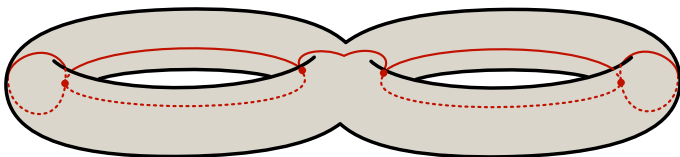


Precise driving by **geometry**

Distance

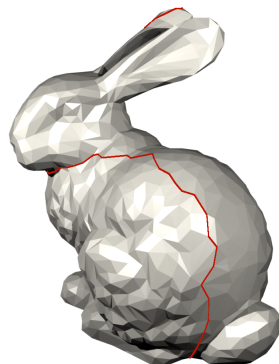
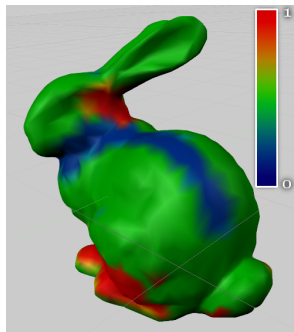
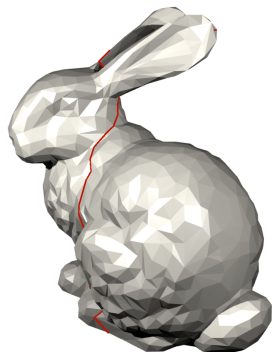
Topological and geometrical **cutting**:

- Length of the **loops** and **paths**
- Depends on **edges' length** (Dijkstra)
- Generally speaking, Euclidean distance: \mathbb{R}^3 : $l_3(e)$



Other choices available

Cutting driven by user map and/or application

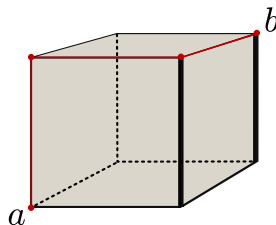
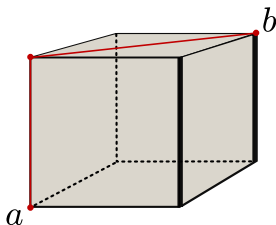


$$m_u : V \rightarrow [0, 1]$$

$$l((v_1, v_2)) = l_3((v_1, v_2)) \times m_u(v_1) \times m_u(v_2)$$

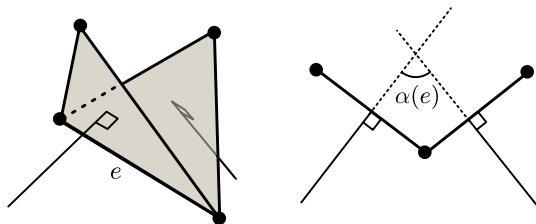
Using local curvature

Constrain the cutting using local **curvature**



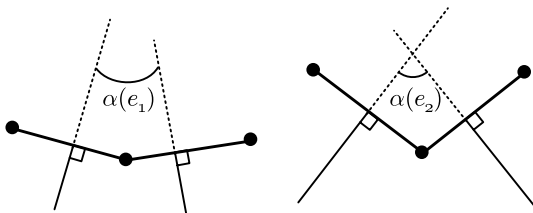
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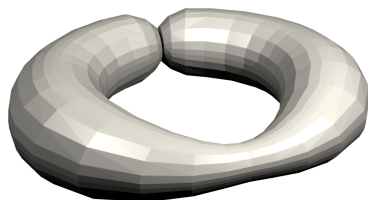
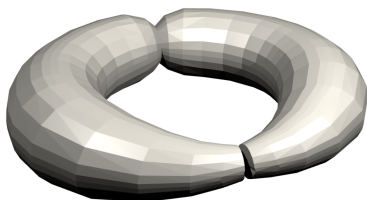


$$l(e) = l_3(e) \times \left(1 - c \frac{|\alpha(e)|}{\pi}\right)$$

with c constant

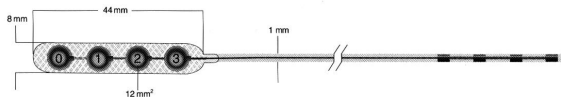
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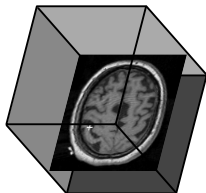
Medical context

Cortical stimulation



Medical context

Cortical stimulation

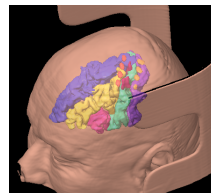
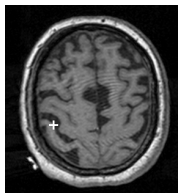


Available data:

- Anatomical MRI

Medical context

Cortical stimulation, electrodes' localization

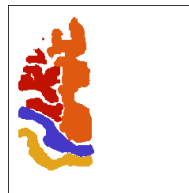
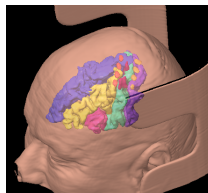
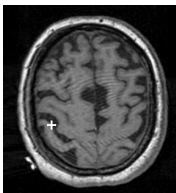


Available data:

- Anatomical MRI
- 3D location of the electrodes

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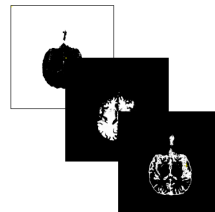
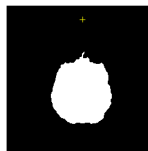
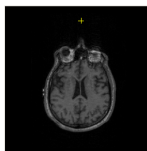


Available data:

- Anatomical MRI
- 3D location of the electrodes
- Various specific information

Preprocessing: from the MRI to the surface

- Segmentation



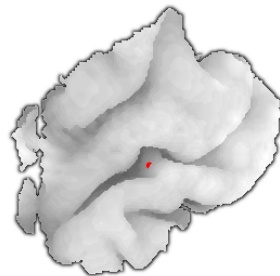
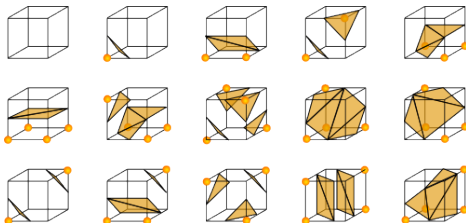
Preprocessing: from the MRI to the surface

- **Segmentation**
- Computation of a brain **mask**



Preprocessing: from the MRI to the surface

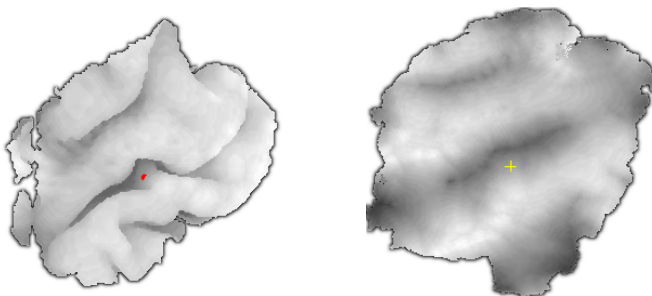
- **Segmentation**
- Computation of a brain **mask**
- Computation of the **surface**



Flat map

Visualization tool: **flat map**

- Topological cutting (genus: 0, boundary: 1)
- Unfolding (conformal or quasi-conformal approaches)



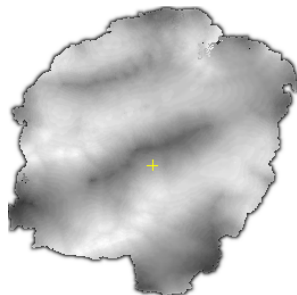
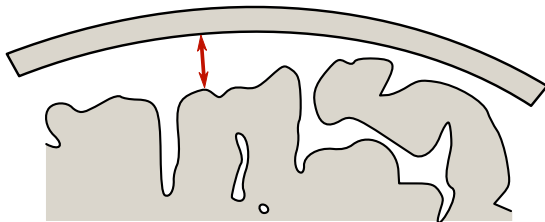
C. R. Collins and K. Stephenson, *A Circle Packing Algorithm*, Computational Geometry: Theory and Applications, 2003

R. Zayer, B. Lévy and H.-P. Seidel, *Linear Angle Based Parameterization*, ACM/EG Symposium on Geometry Processing, 2007

Flat map

Visualization tool: **flat map**

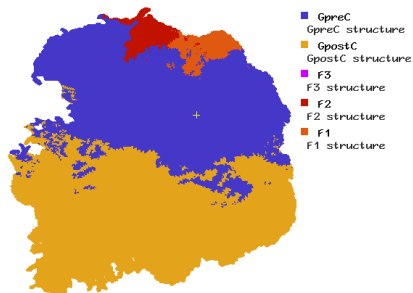
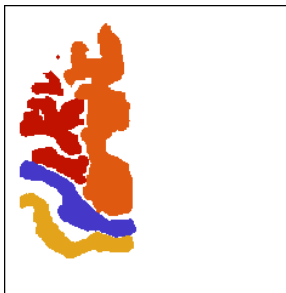
- Topological cutting (genus: 0, boundary: 1)
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- **Data projection**



Flat map

Visualization tool: **flat map**

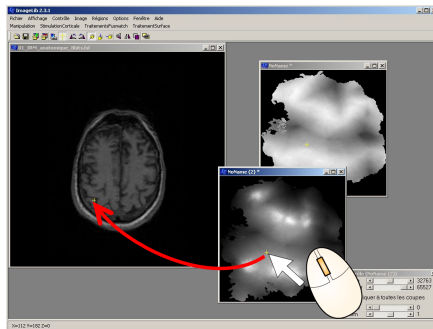
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Flat map

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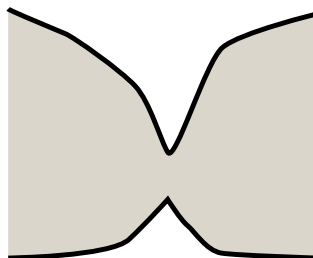
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Limitations

Topological errors:

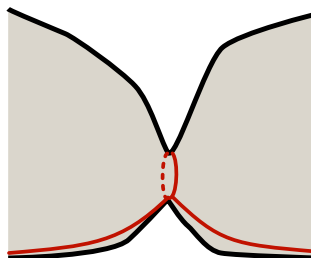
- **Partial volume** effects
- Imprecise **segmentation**



Limitations

Topological errors:

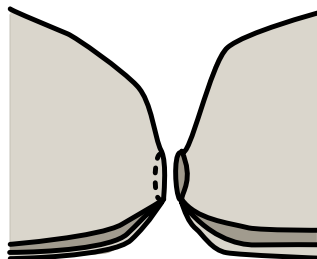
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Limitations

Topological errors:

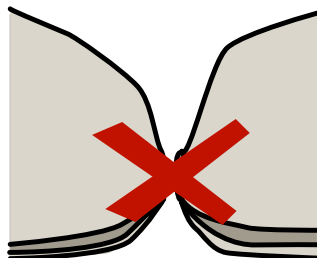
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Limitations

Topological errors:

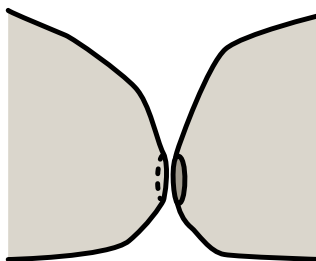
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Correcting the α -junctions

After computation of the shortest loop

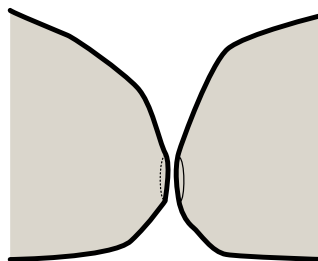
- **Cut the 2-mesh** according to the loop /



Correcting the α -junctions

After computation of the shortest loop

- **Cut the 2-mesh** according to the loop l
- If $length(l) < \alpha$: α -junction detection
 - Add 2 **small discs** to close the cutting



Correcting the α -junctions

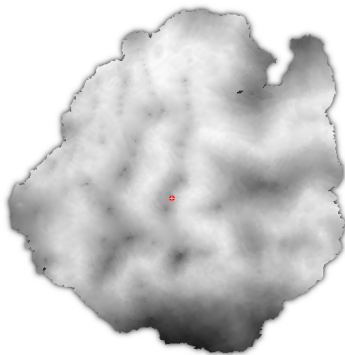
Imprecise segmentation \Rightarrow broken maps



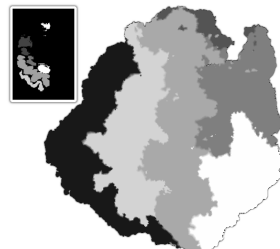
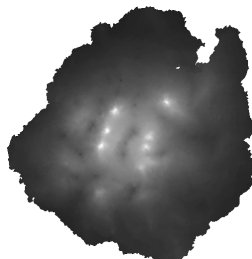
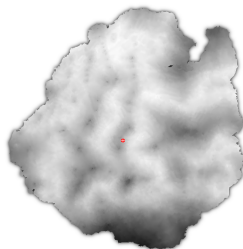
Correcting the α -junctions

Imprecise segmentation \Rightarrow broken maps

Correcting the α -junctions



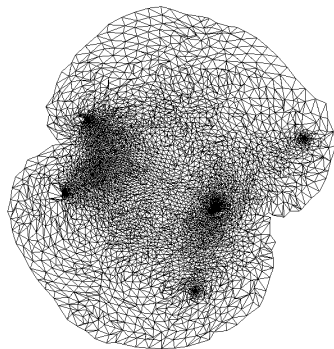
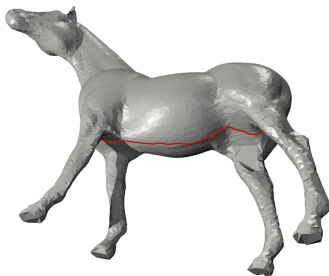
Results



Selecting the extrema

Distortion as geometrical information

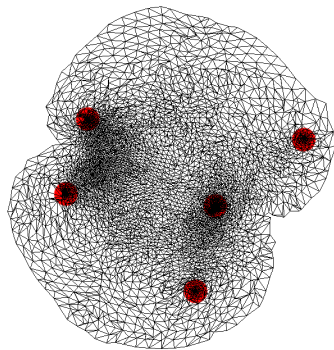
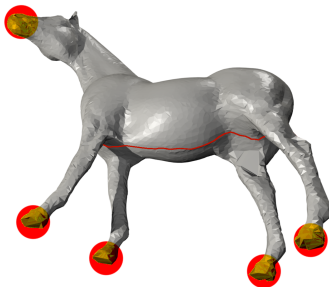
Great distortion of triangles:



Selecting the extrema

Distortion as geometrical information

Great distortion of triangles:



⇒ **Detect compressed areas (protrusions)**

Local distortion

- \mathcal{M} : 2-mesh
- Edge length in \mathbb{R}^3 : $l_3 : E(\mathcal{M}) \rightarrow \mathbb{R}^+$
- Edge length in \mathbb{R}^2 : $l_2 : E(\mathcal{M}) \rightarrow \mathbb{R}^+$

Local distortion

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Definition

The *distortion* of an edge (v_1, v_2) is defined as

$$D(v_1, v_2) = l_3(v_1, v_2) / l_2(v_1, v_2)$$

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Definition

The *distortion* of an edge (v_1, v_2) is defined as

$$D(v_1, v_2) = l_3(v_1, v_2) / l_2(v_1, v_2)$$

Definition

The *distortion* of an vertex v is defined as

$$D(v) = \frac{1}{|(v, v_i) \in E(\mathcal{M})|} \sum_{(v, v_i) \in E(\mathcal{M})} D(v, v_i)$$

Extremum definition

- \mathcal{M} : 2-mesh and B its boundary
- Using $D(\cdot, \cdot)$ in **Dijkstra algorithm** from B

\Rightarrow defining a distance function: L_b

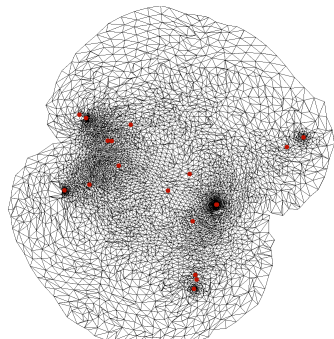
Extremum definition

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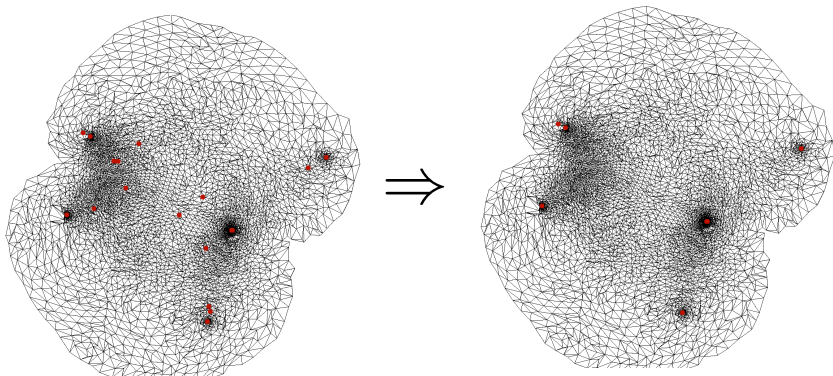
A *extremum* vertex v is a local maximum of L_b



Extremum filtering

For graphical applications: **too many extrema**

- Ordering extrema using $D(\cdot)$
- Keep **only high distortion vertices** (mean, median, manual, etc.)



Geometrical cutting

Data:

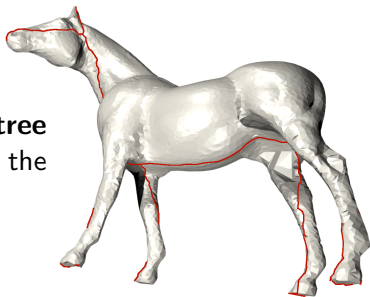
- A 2-mesh \mathcal{M} homeomorphic to a disc
- A set of extrema $(m_i)_{0 \leq i < n}$

Geometrical cutting

Data:

- A 2-mesh \mathcal{M} homeomorphic to a disc
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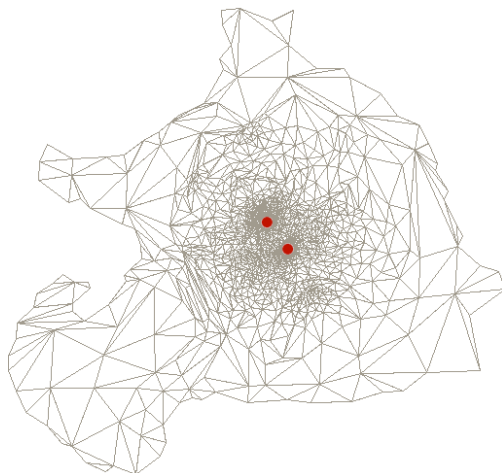
Cut according to the **shortest spanning tree** joining $(m_i)_{0 \leq i < n}$ and the boundary using the **geodesic distance**



Step by step example



Step by step example

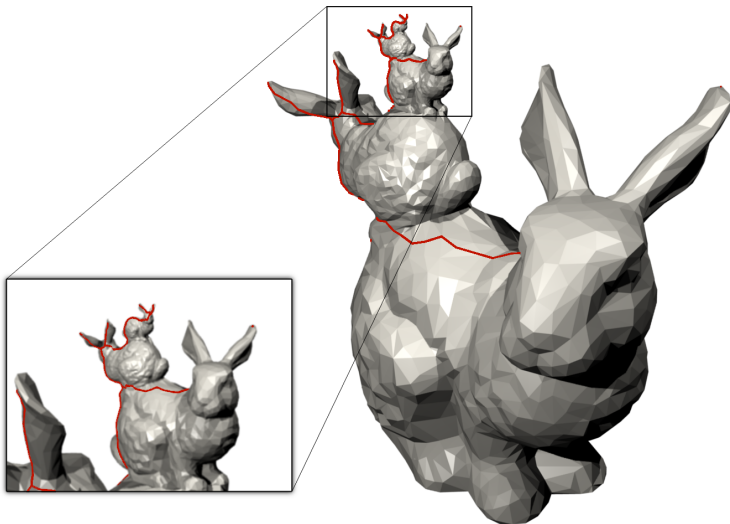


Number of extrema: 19

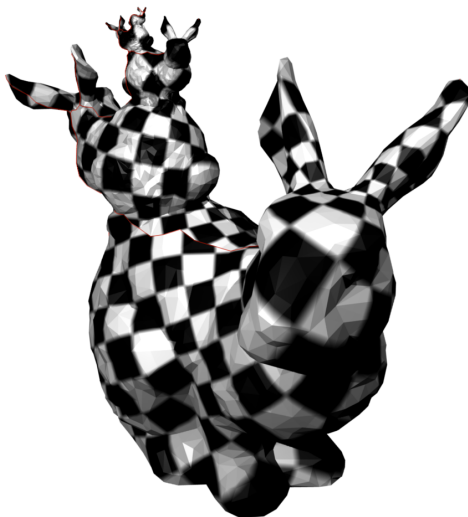
Step by step example



Step by step example

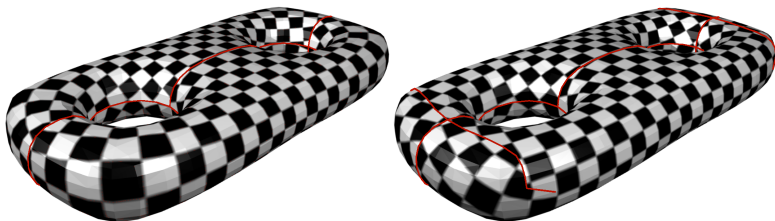


Step by step example



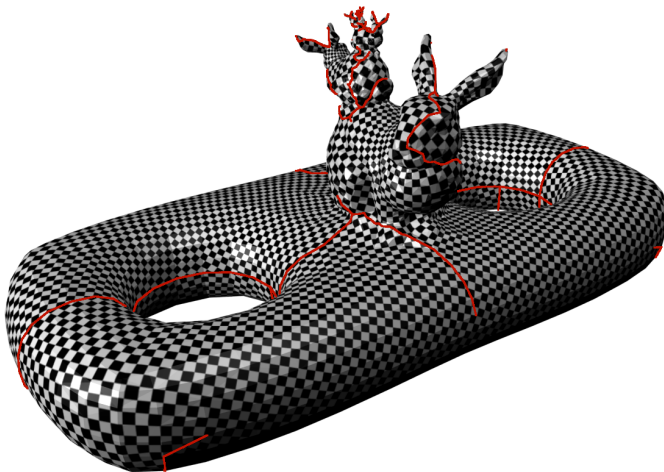
Application: texture mapping

Texture regularity



Application: texture mapping

The multiscale rabbit on a double torus



- 1 Introduction
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 - Shape description
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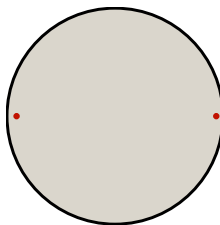


Cylinders



Genus: 0, boundaries: 2

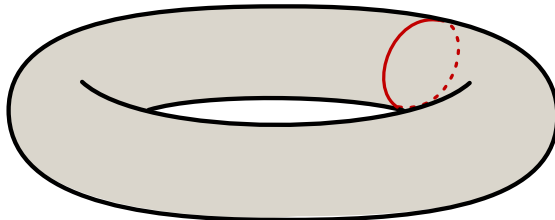
Cylinders



Genus: 1, boundaries: 0



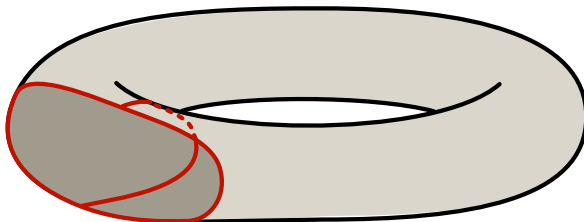
Cylinders



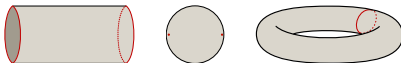
Genus: 1, boundaries: 0



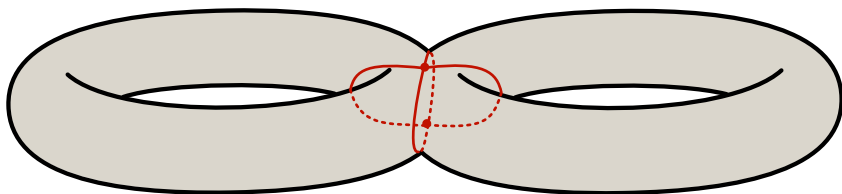
Cylinders



Genus: 1, boundaries: 1



Cylinders

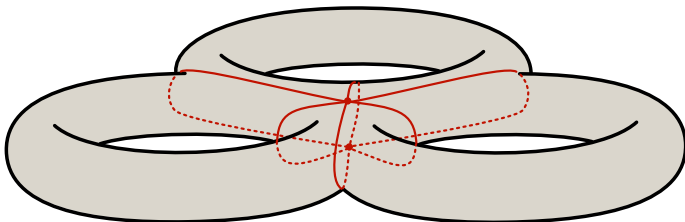


Genus: 2, boundaries: 0

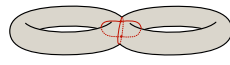
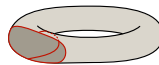
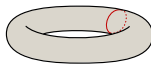


Cutting into cylinders

Cylinders



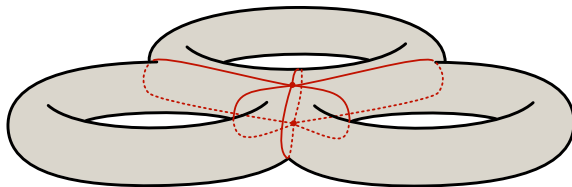
Genus: 3, boundaries: 0



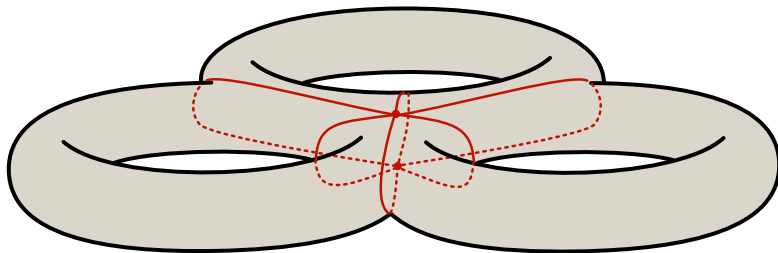
n-loops: definition

Definition (*n*-loop on a 2-mesh \mathcal{M})

- Two **base points** b_0 and b_1
- n non-homotopic **paths** joining b_0 and b_1

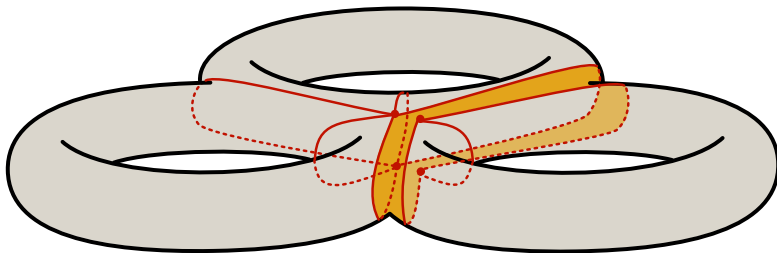


From *n*-loop to 3-loops



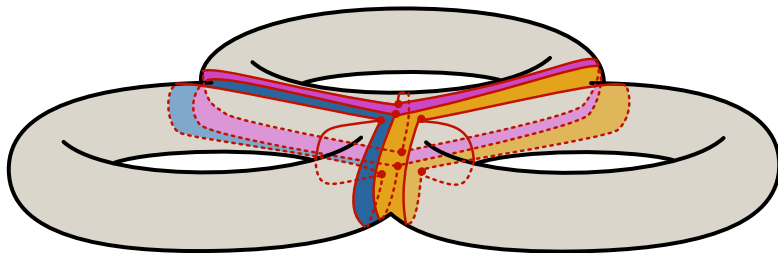
three cylinders, one 6-loop

From *n*-loop to 3-loops



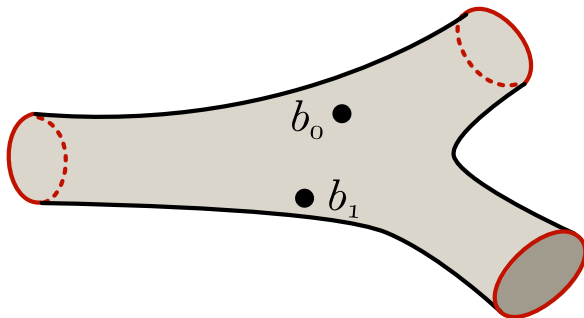
four cylinders, one 5-loop, one 3-loop

From *n*-loop to 3-loops

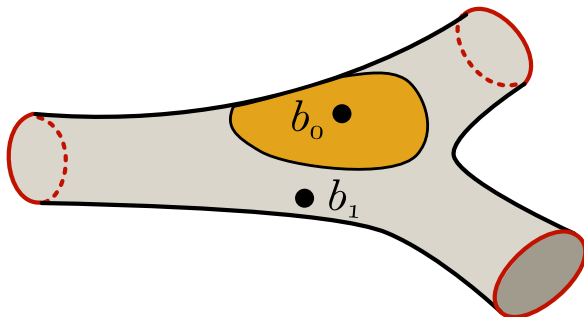


six cylinders, four 3-loop

Approximation of minimal 3-loop

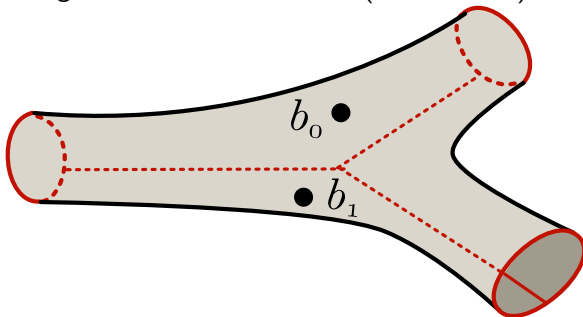


Approximation of minimal 3-loop



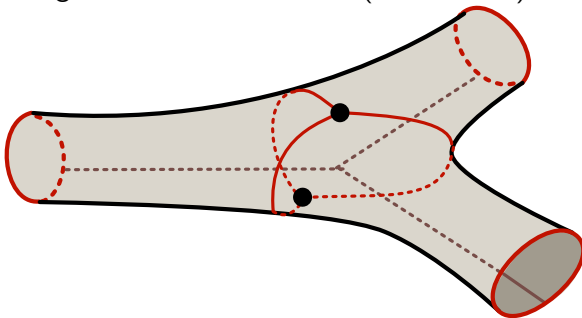
Approximation of minimal 3-loop

Computation using the reduced **cut locus** (Voronoi cell)



Approximation of minimal 3-loop

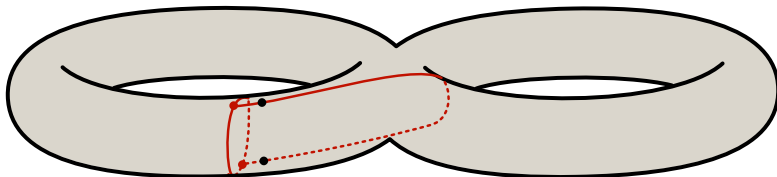
Computation using the reduced **cut locus** (Voronoi cell)



Adjustment by optimization

Goal: segmentation by cylinders using 3-loops

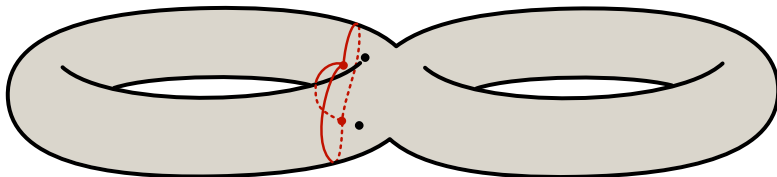
- Principle: **optimize** each 3-loop
- Energy function: **length** of the paths describing the 3-loop



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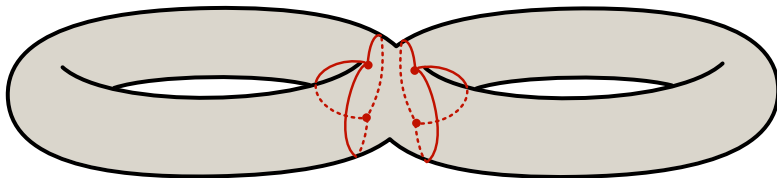
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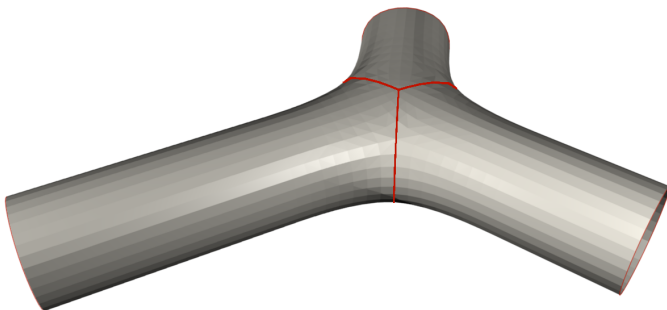
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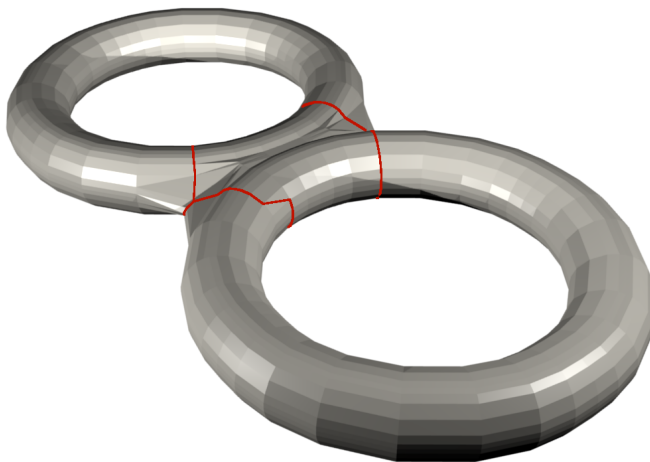
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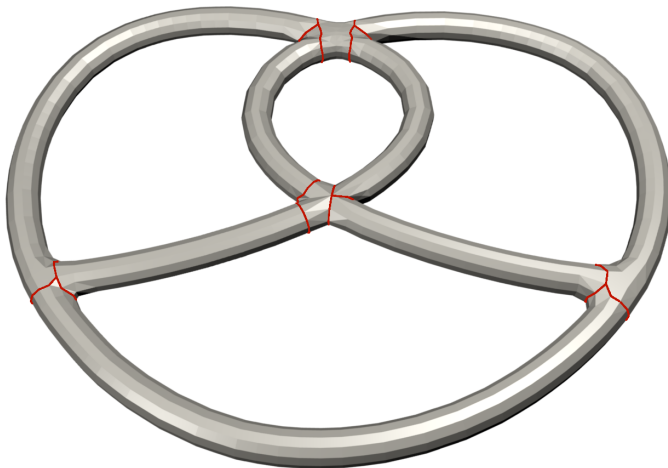
Cylinders from 3-loops: results



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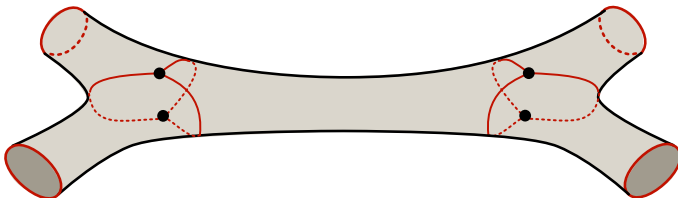


Cylinders from 3-loops: results



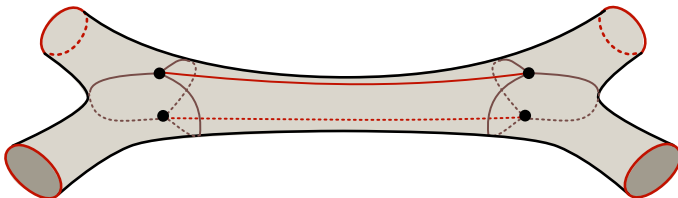
Quadrangulation

Cutting into cylinders by n -loops: tiles with **2 vertices on each boundary**



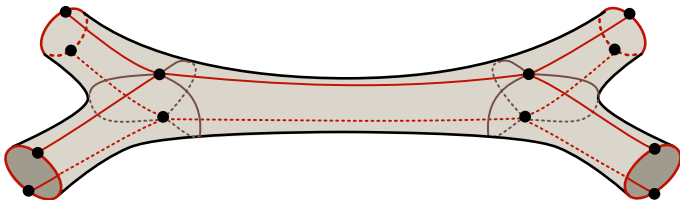
Quadrangulation

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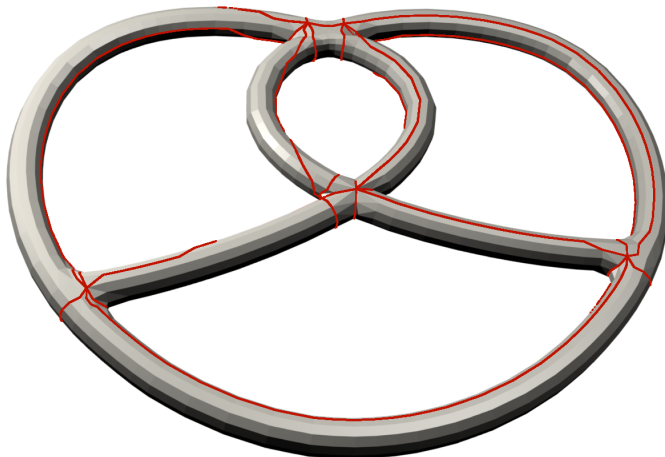
Quadrangulation

Cutting into cylinders by n -loops: tiles with **2 vertices on each boundary**



- Description using **Bézier patches**
- Easy **texture** mapping

Quadrangulation: results



Semantic segmentation

- Removing the detected **extrema**



Semantic segmentation

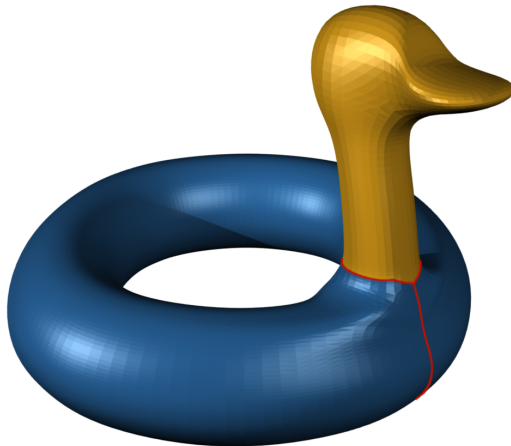
- Removing the detected **extrema**
- Perform a **cutting into cylinders**



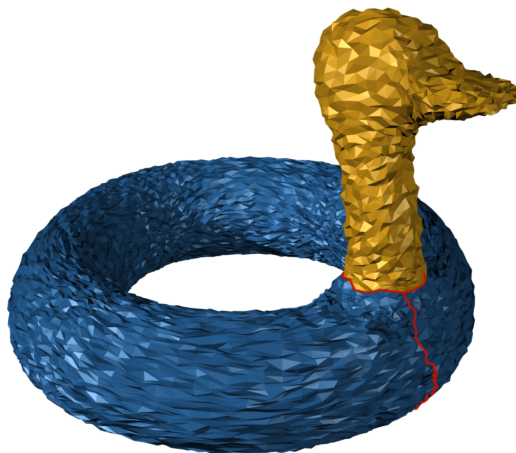
Semantic segmentation: results



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Semantic segmentation: results



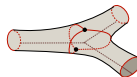
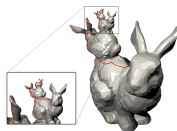
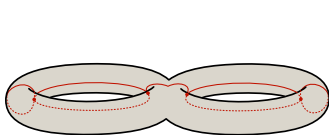
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Conclusion

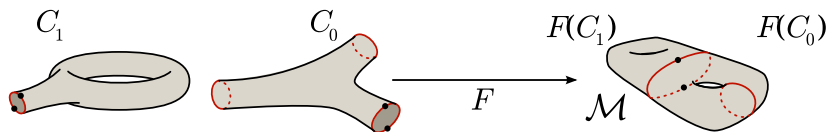
Cutting / tiling:

- Exploit the **topology**
- Integrate the **geometry**
- Supervise the **topology of the tiles**



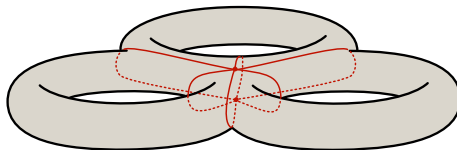
Future

- Expand the using of **M-tilings**



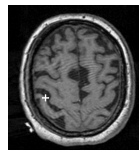
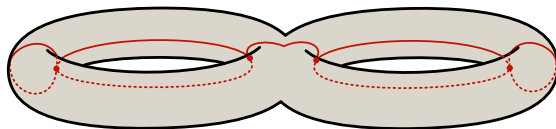
Future

- Expand the using of **M-tilings**
- Explore possibilities of the **n -loops**



Future

- Expand the using of **M-tilings**
- Explore possibilities of the **n -loops**
- Work on the **applications** allowed by **computational topology**



6 Découpage topologique: détails

- Plus court lacet non-séparant
- Améliorations

7 Implementation

- TAGLUT

8 Point clouds

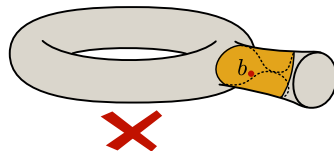
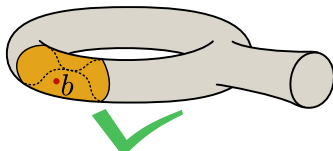
- Geometry



Shortest non-separating loop (1)

Given a **basepoint** b

- Using a “**geodesic**” distance (e.g. Dijkstra¹)
- Circular **wavefront propagation**
- Catching the **junctions** (and their nature)



Complexité: $O(n \log n)$

Dijkstra, A note on two problems in connexion with graphs. *Numerische Mathematik*, 1959.

¹Dijkstra. E. W. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1(1) :269–271, 1959.

Shortest non-separating loop (2)

General method:

- Construct a **set** B potential **basepoints**
- $\forall b \in B$ compute the **shortest** non-separating **loop**
- Keep **the shortest**

Complexité: $O(|B|n \log n)$

Shortest non-separating loop (2)

General method:

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How to compute B ?

Complexité: $O(|B|n \log n)$

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- Keep **the shortest**

How to compute B ?

Principle: a set **crossed by all the non-separating loops**

Complexité: $O(|B|n \log n)$

Shortest non-separating loop (3)

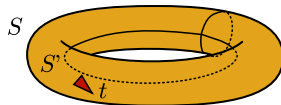
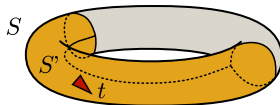
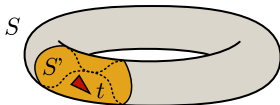
Principle: a set **crossed by all the non-separating loops**

Input: a 2-mesh \mathcal{M} (non homeomorphic to a sphere)

Begin

- Select a starting triangle t
- Add t to a new 2-mesh \mathcal{M}'
- **While** all triangles are not visited **do**
 - Select a non visited triangle t_i adjacent to a visited triangle
 - Stick t_i to its neighbours on \mathcal{M}' (preserving **the genus of \mathcal{M}' : 0**)
- B : set of the boundary points of \mathcal{M}'

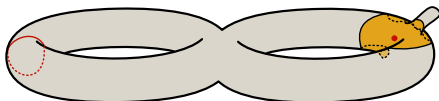
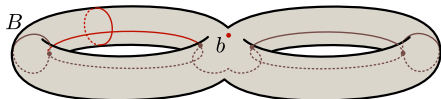
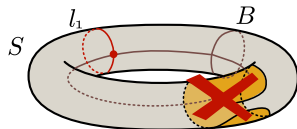
end



Improvements of the Cutting Algorithm

Improvements of the **shortest loop computation**:

- Bounded Dijkstra (using previous cuts)
- Ordering basepoints
- Use minimal length



- 6 Découpage topologique: détails
 - Plus court lacet non-séparant
 - Améliorations

- 7 Implementation
 - TAGLUT

- 8 Point clouds
 - Geometry



TAGLUT

Topological And Geometrical Library : a Useful Toolkit

Informations:

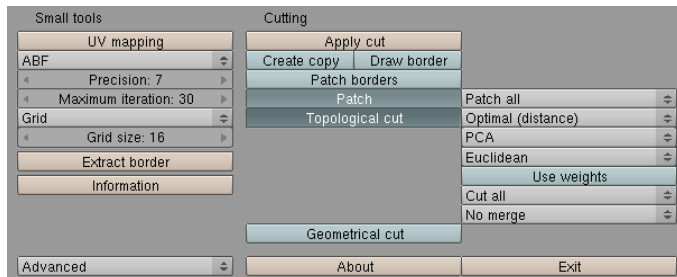
- License: GPL v2 (<http://jmfavreau.info/?q=en/taglut>)
- Languages: C++, python (et xml)
- Supported formats: vrml, ply, obj, etc.

Functionalities:

- Cutting **algorithms** (topology & geometry)
- **Display** and **input/output** functionalities
- Binding with **blender**
- **Scriptable** with python

TAGLUT

blender interface



6 Découpage topologique: détails

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Normals' computation

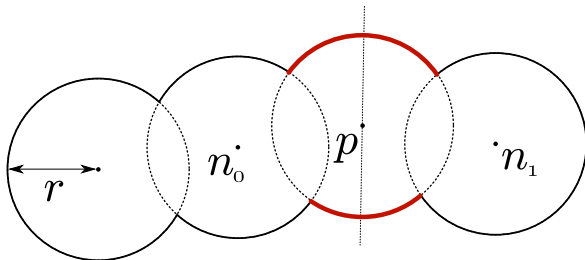
Initial data:

- A sequence of **2D point clouds**
- **Rigid reconstruction** on subsequences
- A sequence of **3D point clouds**



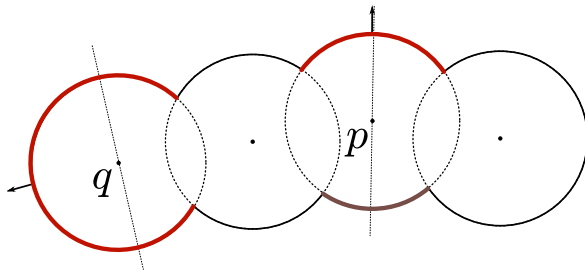
Normals' estimation

- **offset** approach



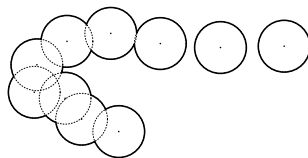
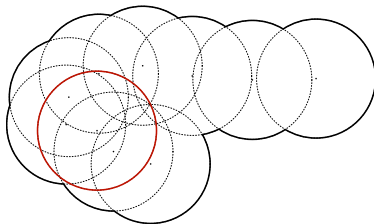
Normals' estimation

- **offset** approach
- Detect the **boundary** property

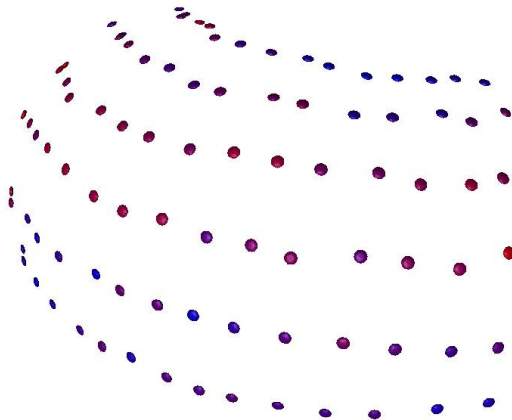


Normals' estimation

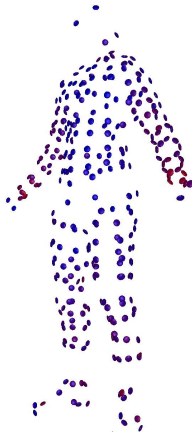
- **offset** approach
- Detect the **boundary property**
- **Local adjustment** of the radii



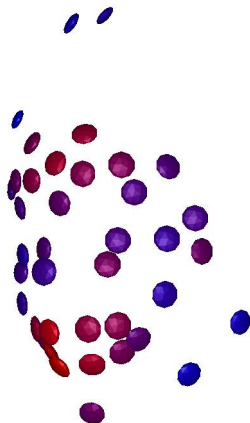
Normals' estimation: results



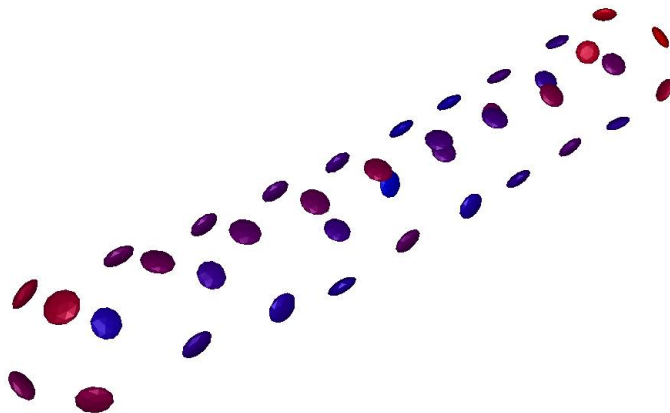
Normals' estimation: results



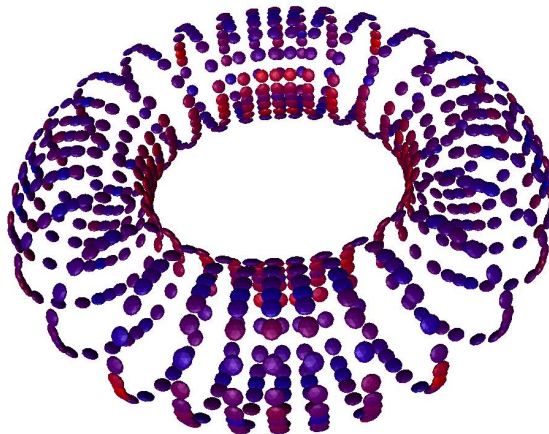
Normals' estimation: results



Normals' estimation: results



Normals' estimation: results



Normals' estimation: results

